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A Study of "Mode Selecting Stochastic Controller" for a Dynamic System Under Random Vibration

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This paper presents a new stochastic controller applied on the vibration control system under irregular disturbances based on the mode selection scheme. Measured displacement and frequency characteristics are simultaneously used in designing a mode selecting controller. This technique is validated by applying to the suppression problem of a flexible beam with randomly vibrated circumstances. The presented method, called Mode Selecting Stochastic Controller, uses the frequency measurement of the flexible system based on a Fast-Fourier transformation algorithm. This controller is constructed by combining mode selection method with previous known Stochastic Controller in real time. Numerical simulations and several experiments are conducted to validate the proposed method. The performance of the proposed method is compared with a stochastic controller developed recently. This method was improved compared with previous one.

Key Words : Stochastic Controller, White Noise, Monte Carlo Method,
F-P-K (Fokker-Plank-Kolomogorv) Equation, Mode Selecting Unit

Nomenclature

L : Length
W : Width
h : Height
ρ : Density
E : Young's Modulus
d : Piezoelectric strain coefficient
$y(x,t)$: Bending displacement
q_i : Generalized modal coordinate
ϕ_i : Mode shape function
ω : Natural frequency
I : Moment of inertia
A : Area
ζ : Damping ratio
$u(t)$: Control input
$d(t)$: Disturbance
N : Data number for frequency analysis
$X(k)$: Discrete Fourier transformation of sampling signal
f_1 : Even order data block of N points frequency analysis data
f_2 : Odd order data block of N points frequency analysis data
F_1 : $\frac{N}{2}$-point DFT result of f_1
F_2 : $\frac{N}{2}$-point DFT result of f_2
F_m : Torque acting on the composite beam
ΔX : Magnitude difference between 1st mode and 2nd mode natural frequency
D_z : PSD of random disturbance

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D_v : PSD of control input

Subscripts

b : Flexible beam
$p.c$: Piezo Ceramic

1. Introduction

Many control systems have complex and nonlinear characteristics, and often, reveals non-stationary processes in nature. Normally, most of them are neglected or simplified in system modeling mainly due to the limitation of system analysis. However, as systems advance users request higher standard of performance, therefore, the neglected characteristics must be taken care of in some ways. These characteristics are significantly affected by unobservable and uncertain parameters of systems. These problems motivated us to generate new and advanced for precise modeling of the system and controller. Several approaches have been tried to solve these problems, and there have been some positive and reliable results.

In this paper, a mode selecting control method is newly proposed and applied to the control of a flexible dynamic system under irregular disturbances. The new approach is applied to the general scheme of a Real Time Stochastic Controller (RTSC) which is developed recently.(Heo et al., 1995 ; 1998 ; 1999 ; 2002 ; 2003) Both natural frequencies and modes are the basic characteristics of system dynamics and essential to the analysis. Thus, real time Mode Selecting Stochastic Controller (MSSC) uses displacement information and frequency characteristics as well. These frequency characteristics are obtained from modal analysis of the flexible dynamic system. In order to use modal characteristics of the flexible beam in real time, MSU (Mode Selecting Unit) has been developed. The primary function of MSU is selecting the dominant mode (1st or 2nd mode) of the system response under irregular disturbances, and secondly it also transfer the information of the selected mode to the controller. The detailed structure and principle of MSU is explained in Chap. III.

Real time version of MSSC is introduced by combining Real Time Stochastic Controller with MSU using modal information of the system in a real time manner. The frequency information of the flexible system is selected from MSU based on an FFT algorithm.

Simulations are performed to predict the performance of the proposed technique. In addition, experiments are conducted to confirm the performance of the controller. A flexible aluminum cantilever beam is excited in a random manner for experimental setup. This cantilever is assumed to be the Euler beam in the numerical analysis. A piezo ceramic element and a laser displacement sensor are used as an actuator and a sensor, respectively. The performance of the proposed technique, MSSC, is compared with that of RTSC.

A conventional control technique using only displacement information, works appropriate without regarding the mode of system under deterministic disturbances. But sometimes it does not work well for the system under indeterminable disturbance conditions. The newly proposed method, MSSC, shows improved results compared with previous one.

The rest of this paper is organized as follows : Section II contains system definition and modeling, Section III presents the controller design method, Section IV contains simulation results, and section V presents experimental results, and section VI concludes with a summary discussion.

2. System

2.1 Modeling system

The following flexible beam in Fig. 1 is adopted as a physical system for the study. Four composite beams in Table 1 are used for experiments

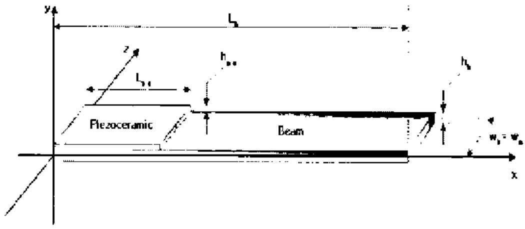


Fig. 1 Physical model of a flexible composite beam with piezo ceramic

Table 1 Specification for experimental model

Parameter	Beam	(No.1)	(No.2)	(No.3)	(No.4)	Piezo ceramic	
Length (m)	L_b	0.45	0.40	0.45	0.40	$L_{p.c}$	0.055
Width (m)	W_b	0.055	0.055	0.055	0.055	$W_{p.c}$	0.055
Thickness (mm)	h_b	0.25	0.25	0.30	0.30	$h_{p.c}$	0.000191
Density (kg/m³)	ρ_b	2700	2700	2700	2700	$\rho_{p.c}$	7700
Young's modulus (N/m²)	E_b	70G	70G	70G	70G	$E_{p.c}$	50G
Piezoelectric strain coefficient (m/V)	—					d_{31}	−180 Pico

to investigate the performance of MSSC according to the beam flexibility in terms of thickness and length.

2.2 Modal analysis

Linear continuous system can be reduced approximately by the mode summation method as in equation (1).

$$y(x,\ t)=\sum_{i}^{\infty}\phi_i(x)\,q_i(t) \tag{1}$$

where

$y(x,t)$: Bending deformation
$\phi_i(x)$: Mode shape function
$q_i(t)$: Generalized modal coordinate

Natural frequencies for each mode for the beam models used in the study are listed in Table 2.

Using the method of weighted residuals such as Galerkin method with the first 2-modes results in practical form of equation as equation (2).

$$\begin{aligned}&\ddot{x}_i(t)+2\zeta_i\omega_i\dot{x}_i(t)+\omega_i x_i(t)\\&=p_i d(t)+b_i u(t), \qquad \text{for } i=1,\ 2\end{aligned} \tag{2}$$

The damping ratio (ζ) in Table 3 is obtained from the free-vibration motion of the composite beam in the experiment.

Table 2 Natural frequency of clamped-free beam

Beam Model	No.1	No.2	No.3	No.4
ω_1	6.7	9.1	9.9	12.8
ω_2	18.7	24.1	26.8	33.3

Table 3 Damping ratios of the beams obtained from experiment

Beam Model	No.1	No.2	No.3	No.4
ζ	0.0068	0.0062	0.0095	0.0091

3. Controller Design

3.1 Mode Selecting Unit (MSU)

The frequency information is extracted from the displacement signal by using a real time mode-selecting unit. For this scheme, a Radix-2 Fast Fourier Transform (FFT) algorithm is employed.

The function of Discrete Fourier Transformation (DFT) for n-point sampling data is

$$X(k)=\sum_{n=0}^{N-1}x(n)\,\omega_N^{kn},\ \text{for } 0\le k\le N-1 \qquad \text{where } \omega_N=e^{j2\pi/N} \tag{5}$$

$\omega_N^2=\omega_{N/2}$, then the DFT equation can be transformed into equation (6)

$$\begin{aligned}X(k)&=\sum_{m=0}^{(N/2)-1}f_1(m)\,\omega_{N/2}^{km}+\omega_N^{k}\sum_{m=0}^{(N/2)-1}f_2(m)\,\omega_{N/2}^{km}\\&=F_1(k)+\omega_N^{k}F_2(k),\ k=0,\ 1,\ \cdots,\ N-1\end{aligned} \tag{6}$$

Where $F_1(k)$ is the DFT result of $f_1(m)$ using N/2 points and $F_2(k)$ is DFT result of $f_2(m)$ using N/2 points. From equation (2), when 1st mode and 2nd mode natural frequency is ω_1, ω_2 and then using equation (7), we can calculate the FFT index of natural frequency

$$k_1=N\frac{\omega_1/2\pi}{f_s},\ k_2=N\frac{\omega_2/2\pi}{f_s} \tag{7}$$

We may round k_1 and k_2 to their respective nearest integers and then compute the difference between the magnitude at 1st mode natural fre-

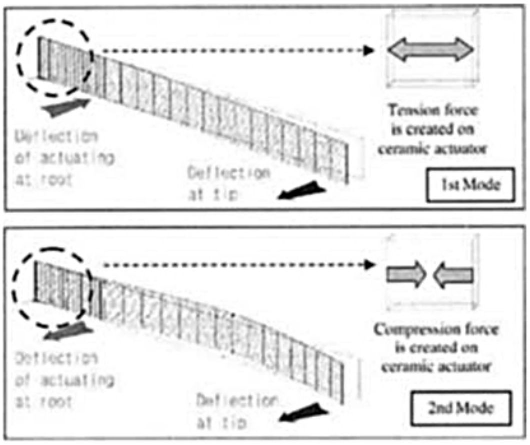


Fig. 2 Actuation on the modes of the flexible cantilever beam

quency and the magnitude at 2nd mode natural frequency.

$$\Delta X = |X(k_{1st})| - |X(k_{2nd})| \tag{8}$$

So using equation (8) we can distinguish the mode of the flexible system. When the sign of ΔX is positive, the 1st mode is dominant for the representation of system motion and when the sign is negative, the 2nd mode is dominant.

The mode shapes of composite cantilever beam are described in the Fig. 2. Due to the difference in mode shape, the directions of actuation at root are assigned according to the direction of each mode shape.

3.2 Mode selecting stochastic controller

The proposed MSSC is composed of MSU module and RTSC (Heo et al., 1999). The design method of RTSC is briefly illustrated in Fig. 3. Detailed design methodology for these kinds of stochastic controller is referred to references (Heo et al., 1998 ; 1999 ; 2002 ; 2003) and (Heo et al., 1995).

Functional block and process can be explained as follows :

(1) This block represents system state equation of the plant in time domain

(2) This process converts from system state equation defined in time domain to dynamic moment equation in stochastic domain

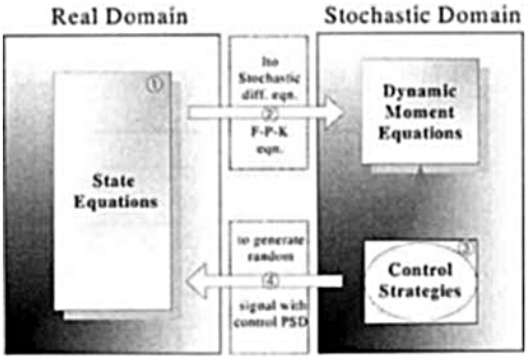


Fig. 3 Basic Concept of RTSC

(3) This block represents the controller for dynamic moment equation

(4) This process is actually the inverse of (2), i.e., the transformation from control signal generated in stochastic domain to the control input in time domain

The governing equation (2) of the flexible beam is transformed to a dynamic moment equation (9) by means of F-P-K method.

$$\dot{m} = A_m m + P_m D_z + B_m D_v \tag{9}$$

where $\dot{m} = [m_{10}\ m_{01}\ m_{11}\ m_{20}\ m_{02}]^T$,

$$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 \\ 0 & 0 & -2\xi\omega_n & -\omega_n^2 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 2\omega_n^2 & 0 & -4\xi\omega_n \end{bmatrix}$$

$$P_m = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix},\ B_m = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}$$

D_z: PSD of random disturbance
D_v: PSD of control input

In designing a controller in the stochastic domain, most of the controller design techniques used in the time domain can be applied. In this paper PID controller is designed and applied. Finally, the obtained control signal (in PSD value) in the stochastic domain is transformed to the control input in the time domain. The control input can be generated by the random signal algorithm

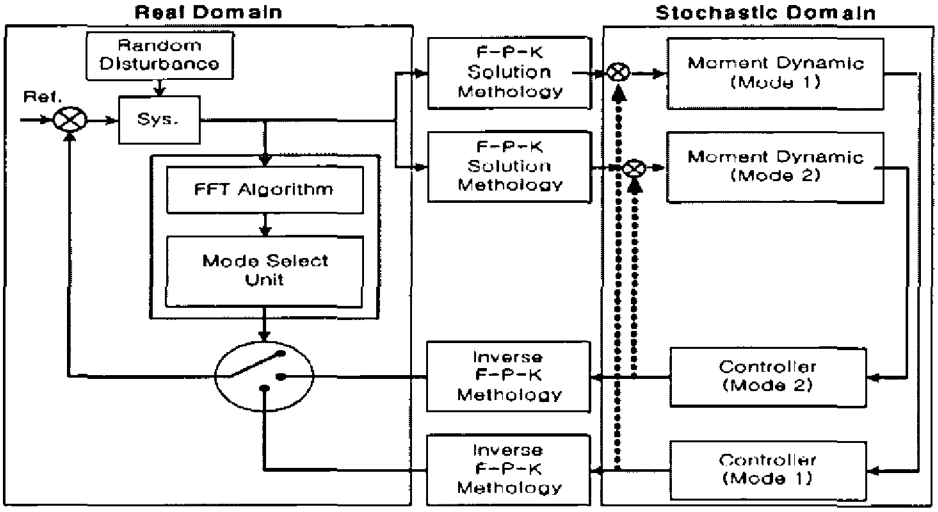


Fig. 4 Conceptual diagram of MSSC

such as Monte-Carlo method (Hellekalek, 1995; Ibrahim, 1985; Nigam and Narayanon, 1994). In case of MSSC design, control signal is generated by inverse F-P-K method (Lee, 2002).

Fig. 4 is a schematic diagram that shows the concept of the proposed MSSC. The basic concept of MSSC is that MSU module is incorporated into RTSC Structure. The design procedure of MSSC is as follows. According to the RTSC design procedure as above, the RTSC for each mode (the 1st mode, the 2nd mode, ⋯, the nth mode) is designed independently. In this study, the MSU module is incorporated in terms of only two-mode approximation.

In the proposed controller, MSU selects only one predominant mode at one time depending on the magnitude of FFT response in each natural frequency.

The direction of tip movement is measured from the laser sensor placed near the tip of beam. The sign of the direction can be regarded as positive or negative according to the tip displacement relative to laser sensor.

Also actuating signal for corresponding selected mode is applied on the root as follows.

If MSU selects the 1st mode as a dominant one, in order to suppress the 1st mode induced system response, actuating direction at the root is implied in opposite sense to make the tip deflection be in neutral position.

Also when MSU selects the 2nd mode as a dominant one, actuating direction at the root is set to same direction of tip deflection due to the excited 2nd mode. The actuating direction is assigned to straighten the deflected shape of beam due to 2nd modal response.

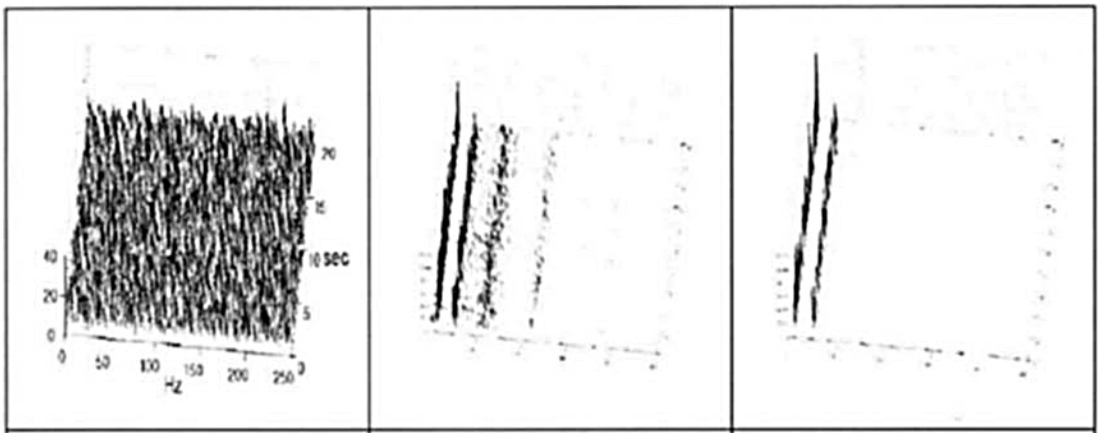
MSU selects only one mode at a time in controlling the system continuously. In order to suppress the response of the beam under random disturbance, control signal is being switched continuously by MSU depending on predominant mode.

4. Simulation

In general, the total response of a system is mixed in terms of each mode (1st mode, 2nd mode, ⋯ and nth mode) of the systems. In this study a two-mode approximated system is adopted to compare with a real physical system. Each system is experiencing the same degree of irregular disturbance, which is white noise (PSD=0.0007) and the system responses are shown in Fig. 5. As can be seen in Fig. 5, the 3rd and 4th modal responses of the real physical system are small comparing with 1st and 2nd modal responses. So a flexible beam used in numerical simulation can be approximated with first two-modes.

Numerical simulation is conducted to verify the proposed concept and the performance of the proposed Mode Selecting Stochastic Controller. One of the important purpose of numerical simulation is to verify whether MSU is engaged very well or not at proper time. And it is also checked possibility of applying MSU to RTSC during simulation.

As shown in Fig. 2, although the directions of the tip deflection of 1st and 2nd modal responses are same, the direction of central deflection for each mode can be opposite. Accordingly, in order to suppress the disturbance quickly and effectively the control signal for actuating on root of beam



3D spectrum | Real physical system | Two-mode approximation

Fig. 5 Spectrum of the system response under the same irregular disturbance

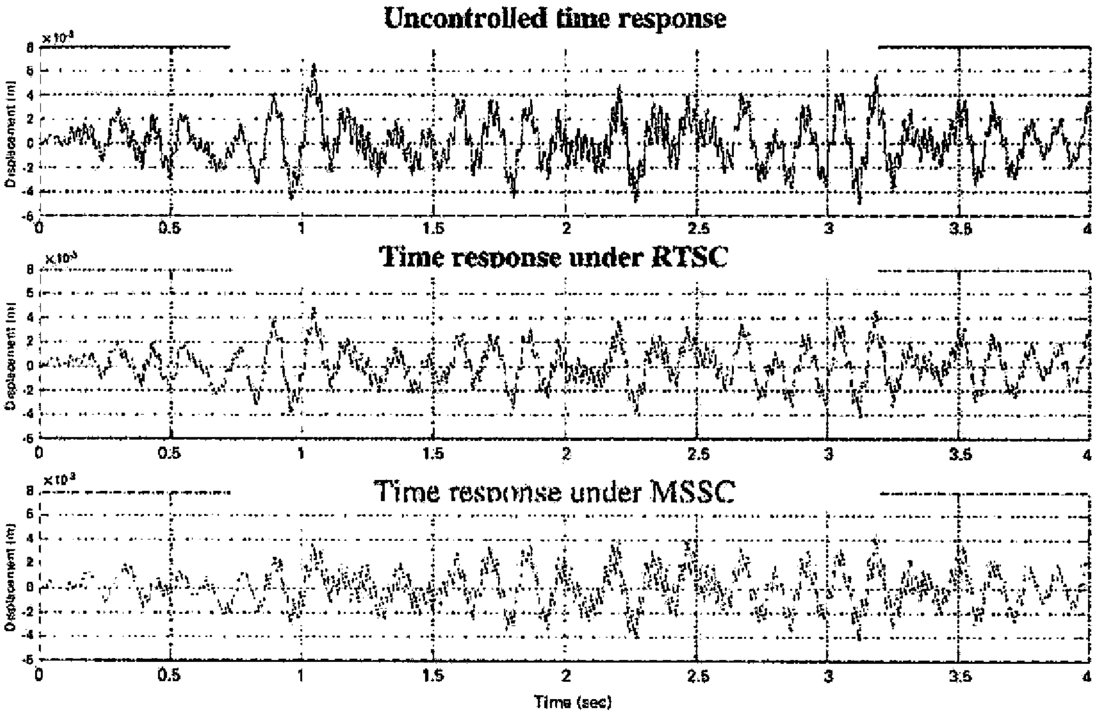


Fig. 6 Comparison of time response

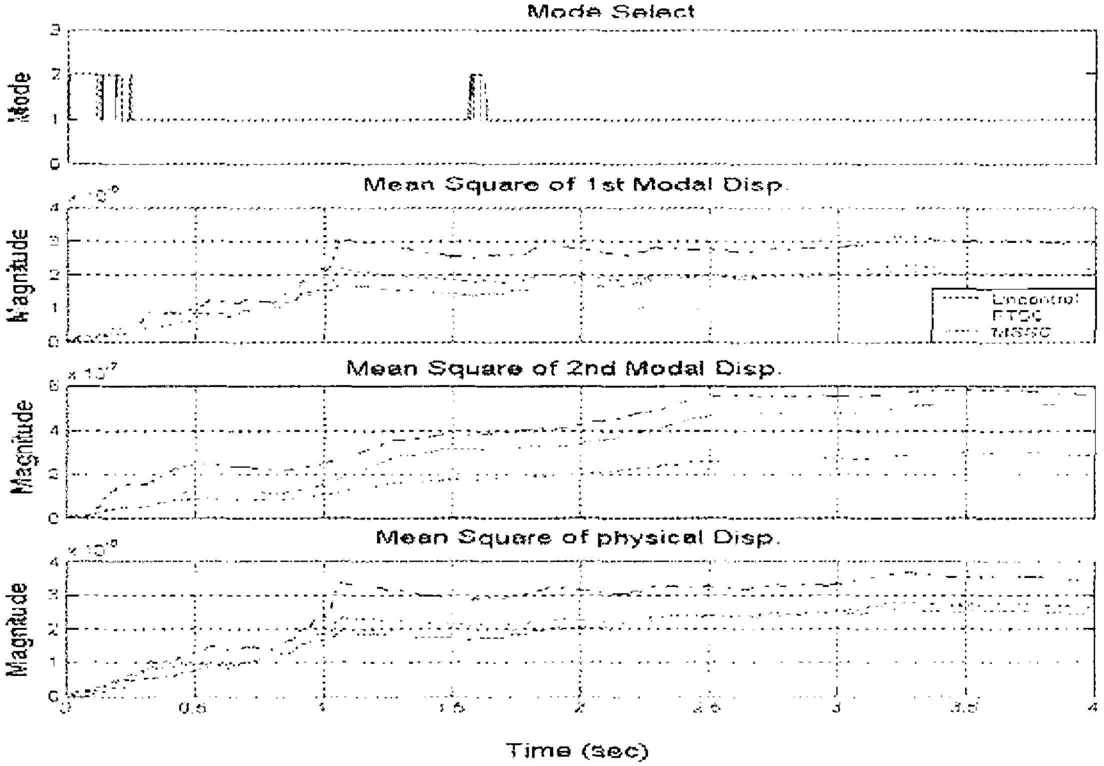


Fig. 7 Comparison of mean square response

is generated in appropriate directions for each mode.

Some discriminative criteria are adopted in the simulation and are conducted in following manner. When the maximum deflection due to the 2nd modal response is greater than half of the *maximum deflection due to the 1st modal response*, then the 2nd mode controller is engaged. In the simulation, the MSU is engaged when the 2nd modal response is greater than 90% of the 1st modal response, which are shown in Fig. 6 and Fig. 7. Results using MSSC are compared with those using RTSC in Fig. 6 and Fig. 7.

The performance of MSSC is demonstrated to be better than that of the RTSC, which uses only displacement information without MSU.

5. Experiment

5.1 Experiment setup

The piezo ceramic, the Piezo System's, PSI-5A-S3, is used as an actuator. Disturbance is generated using a Mini shaker, B&K BE0515. Also a National Instrument, NI-6024 is used for processing input signal measurement and output signals generation. (Fig. 8)

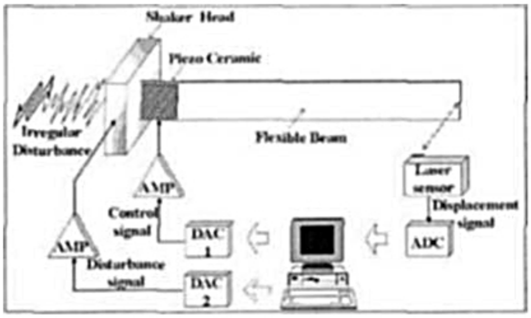


Fig. 8 Schematic diagram of experiment

5.1 Controller performance

To verify the performance of controller, experiments are performed for beams under the same degree of irregular disturbance on their root. In the experiment 4 types of beams illustrated in Table 1 are used to investigate the performance of MSSC according to the beam flexibility in terms of thickness and length. Fig. 9～12 show the experimental results. The mode selecting information of MSSC and responses for the system are illustrated in the first diagram of Fig. 9～12. The second diagram of figure shows the time response of MSSC and uncontrolled system. Also mean square response of MSSC, RTSC and uncontroll-

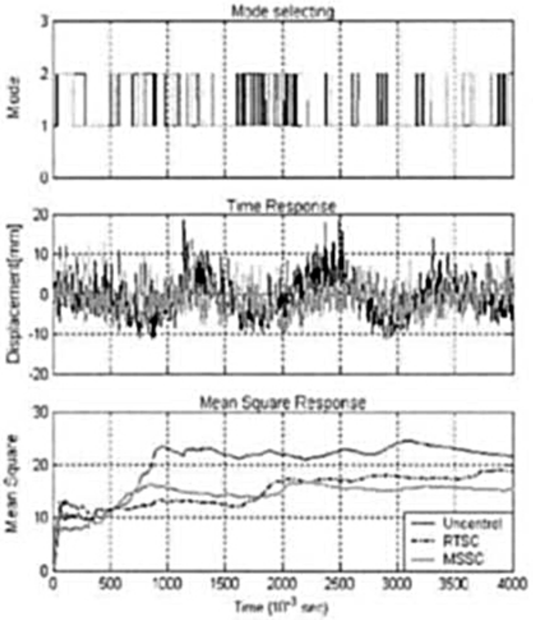


Fig. 9 Comparison of response for the beam no. 1

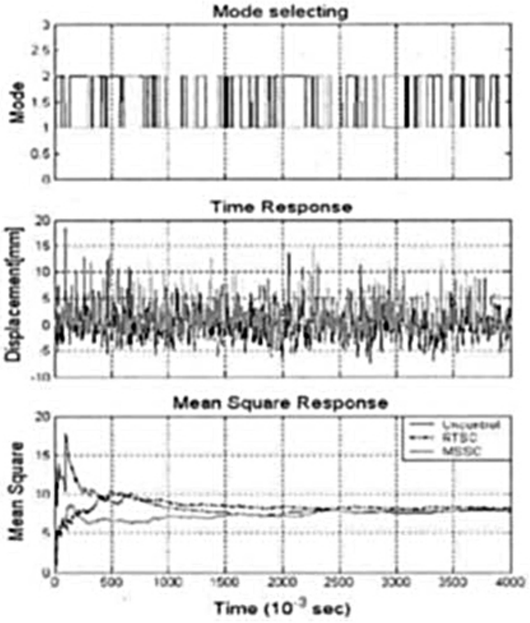


Fig. 10 Comparison of response for the beam no. 2

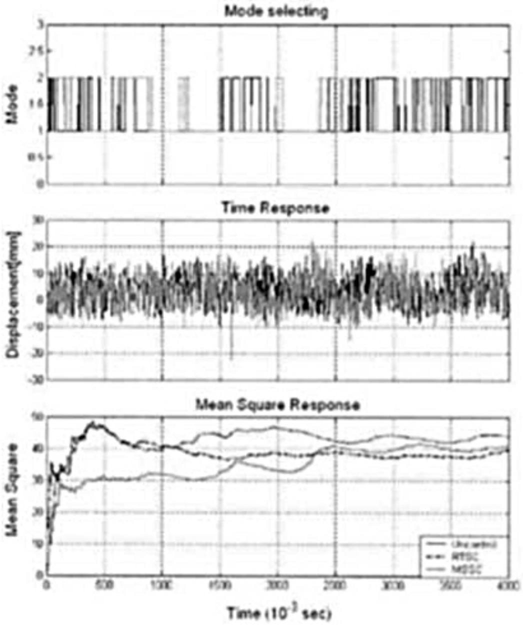


Fig. 11 Comparison of response for the beam no. 3

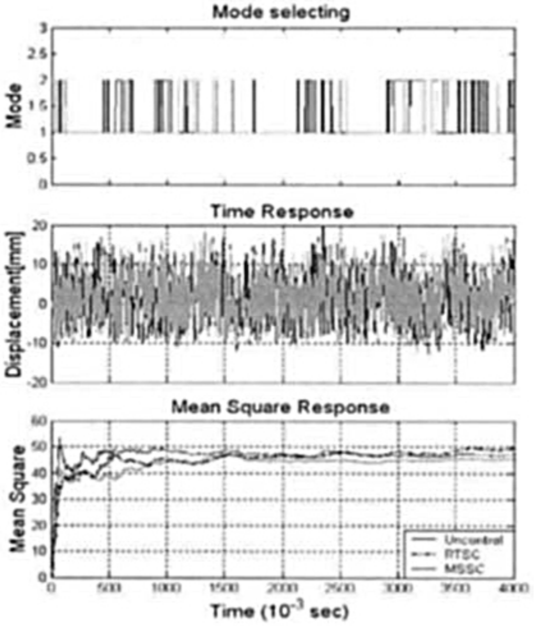


Fig. 12 Comparison of response for the beam no. 4

ed system are shown in the last diagram of each Fig. 9～12.

The MSSC performs generally better in disturbance suppression than the RTSC.

Considering the influence of beam geometric parameters, the proposed mode selecting stochastic controller performs much better for a thinner beam with same length (the controller performance of beam No. 1, 2 are much better than those of beam No. 3, 4). Also the proposed controller shows much better disturbance suppression performance for a longer beam with the same thickness (the controller performance of beam No. 1, 3 are much better than those of beam No. 2, 4).

In short the MSSC performs very well for the system with more flexibility.

The performance of the MSSC is compared with that of RTSC that was designed for the same plant. The result of the experiment shows that performance of MSSC is better than that of the RTSC under the same condition.

6. Conclusions

A new stochastic controller applied on vibration control system under irregular disturbances is presented based on the mode selection scheme. In order to design the proposed controller, called Mode Selecting Stochastic Controller (MSSC) in this paper, not only measured displacement information but also frequency characteristics are simultaneously used in designing the controller. This technique is validated by applying to the suppression problem of a flexible beam with randomly vibrated circumstances. Numerical simulations and several experiments are conducted to examine the proposed method. In implementing the controller with flexible dynamic system, clear identification of response direction is one of the indispensable factors in the design of a controller for the dynamic system under external forces. When directional discrepancy of the displacement is detected on a specific position, the level of total system deformation cannot be quantified clearly. Occasionally, for this reason, the controlled response can be worsened in the simulation. This phenomenon is demonstrated and shown in Fig. 2 of section III. This problem is loosened by adapting a MSU, which employes a FFT algorithm in the controller. Even though the system is exposed to high degree of irregular disturbances, the objective system is stabilized with corresponding control input based on the modal information. The performance of MSSC is compared with that of RTSC designed for the same plant. It is demonstrated by simulation and experiment that the MSSC suppresses disturbances more than the RTSC does. The proposed MSSC is adequately applicable to the flexible beam which exhibits more influence on the higher modes. And both spillover and non-minimum phase problem exist inherently in the beam control system, which is very typical phenomena in the control of flexible structural system. It is mainly focused to investigate the performance of proposed control strategy.

In conclusion, the MSSC reveals more improved performance of disturbance suppression than the RTSC does on the same condition.

In the future, one of the next study topics will be the reduction of the calculation time of MSU algorithm.

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